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General $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ Sigma Model with External Sources, Dynamical Breaking and Spontaneous Vacuum Symmetry Breaking

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We give a general $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ sigma model with external sources, dynamical breaking and spontaneous vacuum symmetry breaking, and present the general formulation of the model. It is found that σ and π^0 without electric charges have electromagnetic interaction effects coming from their internal structures. A general Lorentz transformation relative to external sources $J_{\text{gauge}} = (J_{A_\mu}, J_{A_\mu^\kappa})$ is derived, using the general Lorentz transformation and the four-dimensional current of nuclear matter of the ground state with $J_{\text{gauge}} = 0$, we give the four-dimensional general relations between the different currents of nuclear matter systems with $J_{\text{gauge}} \neq 0$ and those with $J_{\text{gauge}} = 0$. The relation of the density's coupling with external magnetic field is derived, which conforms well to dense nuclear matter in a strong magnetic field. We show different condensed effects in strong interaction about fermions and antifermions, and give the concrete scalar and pseudoscalar condensed expressions of σ_0 and π_0 bosons. About different dynamical breaking and spontaneous vacuum symmetry breaking, the concrete expressions of different mass spectra are obtained in field theory. This paper acquires the running spontaneous vacuum breaking value σ_0', and obtains the spontaneous vacuum breaking in terms of the running σ_0', which make nucleon, σ and π particles gain effective masses. We achieve both the effect of external sources and nonvanishing value of the condensed scalar and pseudoscalar paticles. It is deduced that the masses of nucleons, σ and π generally depend on different external sources.

KEY WORDS: sigma model; external sources; dynamical breaking; spontaneous vacuum symmetry breaking.

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1. INTRODUCTION

It is well known that a lot of different unified models of the electroweak and the strong interactions utilized symmetry breaking. Now it is widely believed that the underlying laws of the world have essential symmetries (Weinberg, 1967; Salam, 1968), symmetric equations may have asymmetric solutions, and there also are various symmetry breaking in the world (Anderson, 1958; Nambu, 1960). Dynamical symmetry breaking of extended gauge symmetries is given (Appelquist and Shrock, 2003), Thomas, Melnitchouk and Steffens (2000) showed dynamical symmetry breaking in the sea of the nucleon, and Antusch *et al.* (2003) investigated dynamical electroweak symmetry breaking by means of neutrino condensation.

Spontaneous symmetry breaking plays an important role in constructing the different unified theories of the electroweak and the strong interactions, as well as gravity theory (Migdal and Polyakov, 1967). But the fundamental scalar field, e.g. Higgs particle, has not been discovered up to now, even though the low energy limit of finding Higgs particle has been raised to very high (Acciarri *et al.*, 2000), e.g., in testing the standard model of weak-electric interaction. Different grand unified theories have many parameters adjusted to fit experiment data, which make the theoretical predication to physical properties be decreased. On the other hand, there are other mechanisms of yielding particle masses (Jackiw and Johnson, 1973; Deser *et al.*, 1982; Holdom, 1981; Susskind, 1979). Schwinger (1962a,b) indicated that if the vacuum polarization tensor has a pole at light-like momentum, gauge fields may acquire mass. A classical σ model of chiral symmetry breaking was given in Nambu and Jona-Lasino (1961a,b), and an in-medium QMC model parameterization and quark condensation in nuclear matter were studied in Guo *et al.* (1999, 2000). Gusynin, Miransky and Shovkovy systematically studied spontaneous rotational symmetry breaking and roton like excitations in gauged sigma model at finite density, it is well shown that there exist excitation branches that behave as phonon like quasiparticles for small momenta and as roton like ones for large momenta (Gusynin *et al.*, 2004).

Pure interactions mediated by swapped mesons between fermions and antifermions possibly yield the vacuum condensation of fermion-antifermions pairs (Liu and Li, 1988), which make the vacuum degeneracy appear. Civitarese *et al.* (1999) researched spontaneous and dynamical breaking of mean field symmetries in proton-neutron quasiparticle random phase approximation and the description of double beta decay transitions. And dynamical chiral symmetry breaking in gauge theories with extra dimensions is also well described (Gusynin *et al.*, 2002).

Dynamical electroweak breaking and latticized extra dimensions are shown up (Cheng *et al.*, 2001), using dynamical breaking, one may make fermions and bosons get masses, and may make the free adjusted parameters decrease, even to a dynamical group of one parameter.

When considering the physical effects of a system coming from another system, a general quantitative causal principle must be satisfied (Huang *et al.*, 2004; Huang and Weng, 2005). Using the homeomorphic map transformation satisfying the general quantitative causal principle, Huang *et al.* (2002) solved the hard problem of the non-perfect properties of the Volterra process, the topological current invariants in Riemann-Cartan manifold, and spacetime defects still satisfy the general quantitative causal principle (Yang *et al.*, 1998). This paper illustrates the fact that σ and π^0 though without electric charges but having electromagnetic interaction effects coming from their internal structures are just result satisfying the general causal rule, i.e., the general quantitative causal principle is essential for researching consistency of models.

In generally analyzing vacuum degeneracy, one studies only the degeneracy vacuum state originated from the self-interaction of scalar fields, one usually neglects the vacuum degeneracy originated from the interactions of different fields.

This paper is different from the past researches, e.g., the researches of Shen and Qiu (1991); Spencer and Zirnbauer (2004), it is organized as follows: Sect. 2 gives a general $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ sigma model with external sources, dynamical breaking and spontaneous vacuum symmetry breaking, Sect. 3 studies different condensed effects (or called condensations) about fermions and antifermions, Sect. 4 presents the formulae of different mass spectra about the vacuum breaking and the dynamical breaking, and shows that the general four dimensional relations between the different currents of nuclear matter systems with $J_{\text{gauge}} \neq 0$ and those with $J_{\text{gauge}} = 0$. The last Section is summary and conclusion.

2. A GENERAL $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ SIGMA MODEL WITH EXTERNAL SOURCES, DYNAMICAL BREAKING AND SPONTANEOUS VACUUM SYMMETRY BREAKING

A Lagrangian density of a general σ model with the chiral symmetry $SU(2)_L \times SU(2)_R$ and electromagnetic $U(1)_{EM}$ is

$$\begin{aligned}\mathfrak{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(x)\gamma^{\mu}(\partial_{\mu} - ieA_{\mu})\psi(x) - g\bar{\psi}(x)\Big[\sigma(x) + \frac{i\boldsymbol{\tau}}{2}\cdot(\boldsymbol{\pi}(x)\\ &+\boldsymbol{\pi}^{+}(x))\gamma_5\Big]\psi(x) - \frac{1}{2}(\partial_{\mu}\sigma(x))^2 - \frac{1}{2}(\partial_{\mu} + ieA_{\mu})\boldsymbol{\pi}^{+}\cdot(\partial_{\mu} - ieA_{\mu})\boldsymbol{\pi}\\ &-\frac{\lambda}{4}[\sigma^2(x) + \boldsymbol{\pi}^2(x) - \nu^2]^2 \qquad (1)\end{aligned}$$

where $\psi(x) = \left|\begin{matrix} p(x) \\ n(x)\end{matrix}\right|$, or $\left|\begin{matrix} u(x) \\ d(x)\end{matrix}\right|$ is isospin double state, $\sigma(x)$ is a scalar field, $\boldsymbol{\pi}(x)$ is pseudoscalar field with isospin 1, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, and $\boldsymbol{\pi}(x)^{+} = \boldsymbol{\pi}(x)$

(Lurie, 1968), $\boldsymbol{\tau}$ is Pauli matrix. The system's Lagrangian with external sources is

$$\mathfrak{L}_j = \mathfrak{L} + \bar{\eta}\psi + \bar{\psi}\eta + J_\sigma \sigma + \mathbf{J}_\pi \cdot \pi + J_{A_\mu} A_\mu. \tag{2}$$

Euler-Lagrange Equations of the system are

$$[\gamma^\mu \partial_\mu - ieA_\mu + g(\sigma(x) + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)\gamma_5)]\psi(x) - \eta(x) = \mathbf{0}, \tag{3}$$

$$\bar{\psi}(x)[-\gamma^\mu \overleftarrow{\partial_\mu} - ie\gamma^\mu A_\mu + g(\sigma(x) + i\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)\gamma_5)] - \bar{\eta}(x) = \mathbf{0}, \tag{4}$$

$$(\Box + \lambda\nu^2)\sigma(x) - g\bar{\psi}(x)\psi(x) - \lambda\sigma(x)(\sigma^2(x) + \boldsymbol{\pi}^2(x)) + J_\sigma(x) = \mathbf{0}, \tag{5}$$

$$\begin{aligned}(\Box + \lambda\nu^2)\boldsymbol{\pi}(x) - \mathbf{e}^2 A_\mu^{\mathbf{2}}(x)\boldsymbol{\pi}(x) - \mathbf{g}\bar{\psi}(x)i\boldsymbol{\tau}\gamma_5\psi(x) - \lambda\boldsymbol{\pi}(x)(\sigma^2(x) + \boldsymbol{\pi}^2(x))\\ +J_\pi(x) = \mathbf{0},\end{aligned} \tag{6}$$

and

$$\partial_\nu F^{\mu\nu} + ie\bar{\psi}(x)\gamma^\mu\psi(x) - e^2 A_\mu(x)\boldsymbol{\pi}^2(x) + J_{A_\mu} = \mathbf{0}. \tag{7}$$

Then we have

$$\begin{aligned}\langle\bar{\psi}(x)\gamma^\mu\partial_\mu\psi(x)\rangle_0^J - ie\langle\bar{\psi}(x)A_\mu(x)\psi(x)\rangle_0^J + g\langle\bar{\psi}(x)\sigma(x)\psi(x)\rangle_0^J\\ + i\langle\bar{\psi}(x)\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)\gamma_5\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} - \langle\bar{\psi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\eta(x) = \mathbf{0}\end{aligned} \tag{8}$$

and

$$\begin{aligned}\langle\bar{\psi}(x)\gamma^\mu \overleftarrow{\partial\mu}\ \psi(x)\rangle_0^J + ie\langle\bar{\psi}(x)A_\mu(x)\psi(x)\rangle_0^J - g\langle\bar{\psi}(x)\sigma(x)\psi(x)\rangle_0^J\\ -i\langle\bar{\psi}(x)\boldsymbol{\tau} \cdot \boldsymbol{\pi}(x)\gamma_5\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + \bar{\eta}(x)\langle\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} = \mathbf{0}.\end{aligned} \tag{9}$$

We can further obtain

$$(\Box + \lambda\nu^2)\langle\sigma(x)\rangle_0^J - g\langle\bar{\psi}(x)\psi(x)\rangle_0^J - \lambda\langle\sigma(x)(\sigma^2(x) + \boldsymbol{\pi}^2(x))\rangle_{\mathbf{0}}^{\mathbf{J}} + J_\sigma(x) = \mathbf{0}, \tag{10}$$

$$\begin{aligned}(\Box + \lambda\nu^2)\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} - \mathbf{e}^{\mathbf{2}}\langle A_\mu^{\mathbf{2}}(x)\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} - \mathbf{g}\langle\bar{\psi}(x)i\boldsymbol{\tau}\gamma_5\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} - \lambda\langle\boldsymbol{\pi}(x)(\sigma^{\mathbf{2}}(x)\\ + \boldsymbol{\pi}^{\mathbf{2}}(x))\rangle_{\mathbf{0}}^{\mathbf{J}} + J_\pi(x) = \mathbf{0},\end{aligned} \tag{11}$$

$$\langle\partial_\nu F^{\mu\nu}\rangle_0^J + ie\langle\bar{\psi}(x)\gamma^\mu\psi(x)\rangle_0^J - e^2\langle A_\mu(x)\boldsymbol{\pi}^2(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + J_{A_\mu}(x) = \mathbf{0}, \tag{12}$$

in which for any field, we can define $\langle Y(x)\rangle_0^J \equiv \langle 0_{out}\,|Y(x)|\,0_{in}\rangle_0^J \big/ \langle 0_{out}|\,0_{in}\rangle_0^J$.

The generating functional of the system is

$$Z(J) \equiv \int \left[D\bar{\psi}\right][D\psi][D\sigma][D\pi]\left[DA_\mu\right]\exp\left(\frac{i}{\hbar}\int d^4x\,\mathfrak{L}_J\right) \tag{13}$$

Using the generating function we have

$$\langle Y(x)\rangle_0^J = \hbar \frac{\delta W}{\delta J_Y(x)}, \tag{14}$$

where $Z = e^{iW}$.

On the other hand, using the method of deducing connection Green function from Green function in quantum field theory (Itzykson and Zuber, 1980; Young, 1987) we can have

$$\langle \sigma^3(x)\rangle_0^J = (\langle \sigma(x)\rangle_0^J)^3 + 3\frac{\hbar}{i}\langle \sigma(x)\rangle_0^J \frac{\delta\langle \sigma(x)\rangle_0^J}{\delta J_\sigma(x)} + \left(\frac{\hbar}{i}\right)^2 \frac{\delta^2\langle \sigma(x)\rangle_0^J}{\delta J_\sigma(x)\delta J_\sigma(x)} + \cdots, \tag{15}$$

$$\langle \sigma(x)\boldsymbol{\pi}^2(x)\rangle_{\mathbf{0}}^{\mathbf{J}} = (\langle \boldsymbol{\pi}^2(x)\rangle_{\mathbf{0}}^{\mathbf{J}})^2 \langle \sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + \frac{\hbar}{i}\langle \sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \frac{\delta\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)} + 2\frac{\hbar}{i}\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \cdot \frac{\delta\langle \sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)} + \left(\frac{\hbar}{i}\right)^{\mathbf{2}} \frac{\delta^{\mathbf{2}}\langle \sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)\cdot \delta \mathbf{J}_\pi(x)} + \cdots, \tag{16}$$

$$\langle \boldsymbol{\pi}(x) A_\mu^2(x)\rangle_{\mathbf{0}}^{\mathbf{J}} = (\langle A_\mu(x)\rangle_0^J)^2 \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + \frac{\hbar}{i}\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \frac{\delta\langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_{A_\mu}(x)} + 2\frac{\hbar}{i}\langle A_\mu(x)\rangle_0^J \frac{\delta\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_{A_\mu}(x)} + \left(\frac{\hbar}{i}\right)^2 \frac{\delta^2\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_{A_\mu}(x)\delta J_{A_\mu}(x)} + \cdots, \tag{17}$$

$$\langle A_\mu(x)\boldsymbol{\pi}^2(x)\rangle_{\mathbf{0}}^{\mathbf{J}} = \langle A_\mu(x)\rangle_0^J (\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}})^{\mathbf{2}} + \frac{\hbar}{i}\langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \frac{\delta\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)} + 2\frac{\hbar}{i}\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \cdot \frac{\delta\langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)} + \left(\frac{\hbar}{i}\right)^{\mathbf{2}} \frac{\delta^2\langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)\cdot \delta \mathbf{J}_\pi(x)} + \cdots, \tag{18}$$

the above equations are some power expansions in mean fields about the little quantity$\hbar$, which is essential for researching the physics of different power series about $\hbar$. Because there are possible condensations of $\langle \bar{\psi}(x)\psi(x)\rangle_0^J$, $\langle \bar{\psi}(x) i\tau\gamma_5\psi(\mathbf{x})\rangle_{\mathbf{0}}^{\mathbf{J}}$ and $\langle \bar{\psi}(x)\gamma_\mu\psi(x)\rangle_0^J$ in Eqs. (10–12), respectively, we have

$$\langle \bar{\psi}(x)\sigma(x)\psi(x)\rangle_0^J = \langle \sigma(x)\rangle_0^J \langle \bar{\psi}(x)\psi(x)\rangle_0^J + \frac{\hbar}{i}\frac{\delta\langle \bar{\psi}(x)\psi(x)\rangle_0^J}{\delta J_\sigma(x)} + \cdots, \tag{19}$$

$$\langle \bar{\psi}(x) i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_5\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} = \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \cdot \langle \bar{\psi}(x) i\boldsymbol{\tau}\gamma_5\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + \frac{\hbar}{i}\frac{\delta\langle \bar{\psi}(x) i\boldsymbol{\tau}\gamma_5\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_\pi(x)} + \cdots, \tag{20}$$

$$\langle\bar{\psi}(x)A_\mu(x)\gamma_\mu\psi(x)\rangle_0^J = \langle A_\mu(x)\rangle_0^J\langle\bar{\psi}(x)\gamma_\mu\psi(x)\rangle_0^J + \frac{\hbar}{i}\frac{\delta\langle\bar{\psi}(x)\gamma_\mu\psi(x)\rangle_0^J}{\delta J_{A_\mu}(x)} + \cdots. \tag{21}$$

Hence, we obtain

$$\begin{aligned}
&\langle\bar{\psi}(x)\gamma^\mu\partial\mu\psi(x)\rangle_0^J - ie\langle A_\mu(x)\rangle_0^J\langle\bar{\psi}(x)\gamma^\mu\psi(x)\rangle_0^J + g\langle\sigma(x)\rangle_0^J\langle\bar{\psi}(x)\psi(x)\rangle_0^J \\
&\quad + ig\langle\pi(x)\rangle_0^J\langle\bar{\psi}(x)\tau\gamma_5\psi(\mathbf{x})\rangle_{\mathbf{0}}^{\mathbf{J}} \\
&\qquad - \langle\bar{\psi}(x)\rangle_0^J\eta(x) - e\hbar\frac{\delta\langle\bar{\psi}(x)\psi(x)\rangle_0^J}{\delta J_{A_\mu}(x)} \\
&\qquad\quad + g\frac{\hbar}{i}\frac{\delta\langle\bar{\psi}(x)\psi(x)\rangle_0^J}{\delta J_\sigma(x)} + g\hbar\frac{\delta\langle\bar{\psi}(x)\psi(x)\rangle_0^J}{\delta\mathbf{J}_\pi(x)} + \cdots = 0,
\end{aligned} \tag{22}$$

$$\begin{aligned}
&-\langle\bar{\psi}(x)\gamma^\mu\overleftarrow{\partial\mu}\psi(x)\rangle_0^J - ie\langle A_\mu(x)\rangle_0^J\langle\bar{\psi}(x)\gamma^\mu\psi(x)\rangle_0^J + g\langle\sigma(x)\rangle_0^J\langle\bar{\psi}(x)\psi(x)\rangle_0^J \\
&\quad + ig\langle\pi(x)\rangle_0^J\langle\bar{\psi}(x)\tau\gamma_5\psi(\mathbf{x})\rangle_{\mathbf{0}}^{\mathbf{J}} \\
&\qquad - \bar{\eta}(x)\langle\psi(x)\rangle_0^J - \mathrm{e}\hbar\frac{\delta\langle\bar{\psi}(x)\psi(x)\rangle_0^J}{\delta J_{A_\mu}(x)} + g\frac{\hbar}{i}\frac{\delta\langle\bar{\psi}(x)\psi(x)\rangle_0^J}{\delta J_\sigma(x)} \\
&\qquad + g\hbar\frac{\delta\langle\bar{\psi}(x)\psi(x)\rangle_0^J}{\delta\mathbf{J}_{\boldsymbol{\pi}}(x)} + \cdots = 0,
\end{aligned} \tag{23}$$

and we can have

$$\begin{aligned}
&(\Box + \lambda\nu^2)\langle\sigma(x)\rangle_0^J = g\langle\bar{\psi}(x)\psi(x)\rangle_0^J + \lambda\langle\sigma(x)\rangle_0^J\left[(\langle\sigma(x)\rangle_0^J)^2 + (\langle\boldsymbol{\pi}\left(x\right)\rangle_{\mathbf{0}}^{\mathbf{J}})^{\mathbf{2}}\right] \\
&\quad + \lambda\frac{\hbar}{i}\left[\mathbf{3}\langle\sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\frac{\delta\langle\sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_\sigma(x)} + \langle\sigma(x)\rangle_0^J\frac{\delta\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_{\boldsymbol{\pi}}(x)} + \mathbf{2}\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\cdot\frac{\delta\langle\sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_{\boldsymbol{\pi}}(x)}\right] \\
&\quad + \lambda\left(\frac{\hbar}{i}\right)^{\mathbf{2}}\left[\frac{\delta^{\mathbf{2}}\langle\sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_\sigma(x)\delta\mathbf{J}_\sigma(x)} + \frac{\delta^{\mathbf{2}}\langle\sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_{\boldsymbol{\pi}}(x)\cdot\delta\mathbf{J}_{\boldsymbol{\pi}}(x)}\right] - \mathbf{J}_\sigma(x) + \cdots,
\end{aligned} \tag{24}$$

$$\begin{aligned}
&(\Box + \lambda\nu^2)\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} = \mathbf{g}\langle\bar{\psi}(x)i\boldsymbol{\tau}\gamma_{\mathbf{5}}\psi(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + \lambda\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\left[(\langle\sigma\left(x\right)\rangle_{\mathbf{0}}^{\mathbf{J}})^{\mathbf{2}} + \left(\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\right)^{\mathbf{2}}\right] \\
&\quad + \lambda\frac{\hbar}{i}\left[\mathbf{3}\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\frac{\delta\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_{\boldsymbol{\pi}}(x)} + 2\langle\sigma(x)\rangle_0^J\frac{\delta\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_\sigma(x)} + \langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}\cdot\frac{\delta\langle\sigma(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_\sigma(x)}\right] \\
&\quad + \lambda\left(\frac{\hbar}{i}\right)^{\mathbf{2}}\left[\frac{\delta^{\mathbf{2}}\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_\sigma(x)\cdot\delta\mathbf{J}_\sigma(x)} + \frac{\delta^{\mathbf{2}}\langle\boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta\mathbf{J}_{\boldsymbol{\pi}}(x)\cdot\delta\mathbf{J}_{\boldsymbol{\pi}}(x)}\right] - \mathbf{J}_{\boldsymbol{\pi}}(x)
\end{aligned}$$

$$+e^2 \Bigg[(\langle A_\mu(x)\rangle_0^J)^2 \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} + \frac{\hbar}{i} \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \frac{\delta \langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_{A_\mu}(x)}$$

$$+2\frac{\hbar}{i} \langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \frac{\delta \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_{A_\mu}(x)} + \left(\frac{\hbar}{i}\right)^{\mathbf{2}} \frac{\delta^{\mathbf{2}} \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta J_{A_\mu}(x) \delta J_{A_\mu}(x)} \Bigg] + \cdots, \tag{25}$$

$$\langle \partial_\nu F^{\mu\nu}\rangle_0^J + ie \langle \bar{\psi}(x)\gamma^\mu \psi(x)\rangle_0^J - e^2 \Big[(\langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}})^{\mathbf{2}} \langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}$$

$$+\frac{\hbar}{i} \langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \frac{\delta \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_{\boldsymbol{\pi}}(x)}$$

$$+2\frac{\hbar}{i} \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \cdot \frac{\delta \langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_{\boldsymbol{\pi}}(x)} + \left(\frac{\hbar}{i}\right)^{\mathbf{2}} \frac{\delta^{\mathbf{2}} \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}{\delta \mathbf{J}_{\boldsymbol{\pi}}(x) \cdot \delta \mathbf{J}_{\boldsymbol{\pi}}(x)} + \cdots \Bigg] + J_{A_\mu}(x). \tag{26}$$

And we can further obtain

$$\langle \partial_\mu (\bar{\psi}(x)\gamma^\mu \psi(x))\rangle_0^J = \langle \bar{\psi}(x)\rangle_0^J \eta(x) - \bar{\eta}(x) \langle \psi(x)\rangle_0^J, \tag{27}$$

when $\bar{\eta} = \eta = 0$, it follows that

$$\partial_\mu (\bar{\psi}(x)\gamma^\mu \psi(x)) = 0, i.e., \partial_\mu j^\mu = 0. \tag{28}$$

We neglect the powers with $\hbar$ in the power series, and take external sources into zero, therefore, we deduce

$$g\langle \bar{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0} + \lambda \sigma_0 (\sigma_0^2 + \boldsymbol{\pi}_{\mathbf{0}}^{\mathbf{2}} - \nu^{\mathbf{2}}) = \mathbf{0}, \tag{29}$$

$$ig\langle \bar{\psi}(x)\gamma_5 \tau \psi(\mathbf{x})\rangle_{\mathbf{0}}^{\mathbf{J}} \mid_{\mathbf{J=0}} + \lambda \boldsymbol{\pi}_{\mathbf{0}} (\sigma_{\mathbf{0}}^{\mathbf{2}} + \boldsymbol{\pi}_{\mathbf{0}}^{\mathbf{2}} - \nu^{\mathbf{2}}) = \mathbf{0}, \tag{30}$$

$$\langle \partial_\nu F^{\mu\nu}\rangle_0^J \mid_{J=0} + ie \langle (\bar{\psi}(x)\gamma^\mu \psi(x)\rangle_0^J \mid_{J=0} = 0, \tag{31}$$

where $\sigma_0 = \langle \sigma(x)\rangle_0^J \mid_{J=0}$ and $\boldsymbol{\pi}_{\mathbf{0}} = \langle \pi(\mathbf{x})\rangle_{\mathbf{0}}^{\mathbf{J}} \mid_{\mathbf{J=0}}$.

Analogous to Lurie (1968)'s research, fermion's propagator is

$$\langle \bar{\psi}(x)\psi(x')\rangle_0^J = \frac{1}{(\boldsymbol{\pi})^4} \int^\Lambda \frac{-e^{i(x-x')\cdot p} d^4 p}{\gamma^\mu \cdot p_\mu - ig\langle \sigma(x)\rangle_0^J + g\boldsymbol{\tau} \cdot \langle \boldsymbol{\pi}(x)\rangle_{\mathbf{0}}^{\mathbf{J}} \gamma_{\mathbf{5}} - e\gamma^\mu \langle A_\mu(x)\rangle_{\mathbf{0}}^{\mathbf{J}}}, \tag{32}$$

where Λ is the cutting parameter, Eqs. (28–32) are the basic equations relative to both dynamical breaking and vacuum breaking.

3. DIFFERENT CONDENSATIONS ABOUT FERMIONS AND ANTIFERMIONS AND THE FOUR-DIMENSIONAL GENERAL DIFFERENT CURRENTS

We now generally investigate the different condensations about fermions and antifermions.

When $\sigma_0 \neq 0$, $\langle\bar{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0} \neq 0$, we evidently have

$$\frac{ig\langle\bar{\psi}(x)\gamma_5\boldsymbol{\tau}\psi(x)\rangle_0^J \mid_{J=0}}{g\langle\bar{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}} = \frac{\boldsymbol{\pi}_0}{\sigma_0}, \tag{33}$$

then we generally have

$$\sigma_0 = Kg\langle\bar{\psi}(x)\psi(x)\rangle_0^J \mid_{J=0}, \tag{34}$$

$$\boldsymbol{\pi}_\mathbf{0} = iKg\langle\bar{\psi}(x)\gamma_5\boldsymbol{\tau}\psi(x)\rangle_\mathbf{0}^\mathbf{J} \mid_{\mathbf{J=0}}, \tag{35}$$

where K is the parameter determined by physical experiment data or empirical values from predications of theoretical models. Equation (34) and (35) mean that σ_0 and $\boldsymbol{\pi}_\mathbf{0}$ are directly originated from the dynamical condensations of fermion-antifermion pairs. The condensations also depend on K, which is different from the condensation mechanism before.

Analogous to David and Andre (1974), it shows that under some conditions the fundamental scalar fields are equivalent to the composed scalar fields.

Furthermore, we have

$$ic\langle\rho_e(x)\rangle_0^J \mid_{J=0} = \langle\partial_\nu F^{4\nu}(x)\rangle_0^J \mid_{J=0} = -ie\langle\psi^+(x)\psi(x)\rangle_0^J \mid_{J=0}, \tag{36}$$

$$\langle j_e^i\rangle_0^J \mid_{J=0} = \langle\partial_\nu F^{i\nu}(x)\rangle_0^J \mid_{J=0} = -ie\langle\bar{\psi}(x)\gamma^i\psi(x)\rangle_0^J \mid_{J=0}, \tag{37}$$

where ρ_e and j_e^i are the electric charge density and the electric current density in nuclear matter, respectively. We also may discuss the current by means of the analogous method in Yang *et al.* (1998). Therefore, we obtain the average relation of nuclear matter density and electric charge density at the situation point without external source as follows

$$\rho_g \equiv \langle\rho_B(x)\rangle_0^J \mid_{J=0} = \frac{-c}{e}\langle\rho_e(x)\rangle_0^J \mid_{J=0}, \tag{38}$$

where ρ_g is ground state density of the fermion doublet, and $\rho_B(x) = \psi^+(x)\psi(x)$ is the density operator of the proton and neutron isospin doublet, the physical meaning of Eq. (38) is that the ground state of nuclear density is equal to the condensation of the electric charge density divided by electronic charge and multiplied by $-c$, the condensation is the distribution of the ground state density of charged particles in nuclear matter.

We further get

$$\frac{i}{e}\langle j_e^i\rangle_0^J \mid_{J=0}= \frac{i}{e}\langle \partial_\nu F^{i\nu}(x)\rangle_0^J \mid_{J=0}= \langle j^i\rangle_0^J \mid_{J=0}\equiv j_0^i, \tag{39}$$

where $j^i = \bar{\psi}(x)\gamma^i\psi(x)$ is a vector current density of the nuclear matter.

On the other hand, because, in terms of fundamental interaction principles, the interactions of $U_{EM}(1)$ and $SU_C(3)$ gauge fields generally affect the state of the matter, when the corresponding external sources $J_{\text{gauge}} = (J_{A_\mu}, J_{A_\mu^\kappa}) \neq 0$ (J_{A_μ} and $J_{A_\mu^\kappa}$ are external sources of the interactions of $U_{EM}(1)$ and $SU_C(3)$ gauge fields, respectively, κ is the superscript of $SU_C(3)$ color gauge group), we may generally assume a general equivalent velocity $\mathbf{v}$ (of the nuclear matter system with $J_{\text{gauge}} \neq 0$) relative to the nuclear matter system (of ground state) with $J_{\text{gauge}} = 0$, because the equivalent relative velocity $\mathbf{v}$ is originated from the actions of external sources J_{gauge} with Lorentz subscripts. In fact, the actions of the external sources make the nuclear matter system have the excited equivalently relative velocity $\mathbf{v}$. Therefore, the velocity $\mathbf{v}$ is the function of the external sources, i. e., $\mathbf{v} = \mathbf{v}(\mathbf{J}_{\mathbf{A}_\mu}, \mathbf{J}_{\mathbf{A}_\mu^\kappa}) = \mathbf{v}(\mathbf{J}_{\mathbf{gauge}})$. Using a general Lorentz transformation we can obtain the relations of the four-dimensional general current of nuclear matter system (with $J_{\text{gauge}} \neq 0$) relative to the nuclear matter system (with $J_{\text{gauge}} = 0$) of the ground state as follows

$$\mathbf{j}' = \mathbf{j_0} + \mathbf{v}(\mathbf{J_{gauge}})\left[\left(\frac{1}{\sqrt{1-\frac{\mathbf{v}^2(\mathbf{J_{gauge}})}{\mathbf{c}^2}}}-1\right)\frac{\mathbf{j_0}\cdot\mathbf{v}(\mathbf{J_{gauge}})}{\mathbf{c}^2} - \frac{\rho_{\mathbf{g}}}{\sqrt{1-\frac{\mathbf{v}^2(\mathbf{J_{gauge}})}{\mathbf{c}^2}}}\right], \tag{40}$$

$$\rho' = \frac{\rho_g - \frac{\mathbf{j_0}\cdot\mathbf{v}(\mathbf{J_{gauge}})}{c^2}}{\sqrt{1-\frac{\mathbf{v}^2(\mathbf{J_{gauge}})}{c^2}}}. \tag{41}$$

We, thus, can generally assume the velocity $\mathbf{v}(\mathbf{J_{gauge}})$ linearly depends on the external sources. Therefore, we can obtain a general expression

$$\mathbf{v}(\mathbf{J_{gauge}}) = \alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu} + \alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa} \tag{42}$$

in which $\alpha_{\mathbf{A}_\mu}$ and $\alpha_{\mathbf{A}_\mu^\kappa}$ are the relative coupling constants of external sources J_{A_μ} and $J_{A_\mu^\kappa}$, respectively. Thus, Eqs. (40) and (41) may be rewritten as two general expressions linearly depending on the external sources as follows

$$\mathbf{j}' = \mathbf{j_0} + (\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu} + \alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa})\left\{\left(\frac{1}{\sqrt{1-\frac{(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}+\alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa})^2}{\mathbf{c}^2}}}-1\right)\right.$$
$$\left.\frac{\mathbf{j_0}\cdot(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu} + \alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa})}{c^2} - \frac{\rho_g}{\sqrt{1-\frac{(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}+\alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa})^2}{c^2}}}\right\}, \tag{43}$$

$$\rho' = \frac{\rho_g - \frac{\mathbf{j_0}\cdot(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}+\alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa})}{c^2}}{\sqrt{1-\frac{(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}+\alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa})^2}{c^2}}}. \tag{44}$$

and the consistent condition is

$$|\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu} + \alpha_{\mathbf{A}_\mu^\kappa}\mathbf{J}_{\mathbf{A}_\mu^\kappa}| < c \tag{45}$$

In order to make the theory concrete, we consider a case when the external source $J_{A_\mu^\kappa}$ equates to zero but external J_{A_μ}. Then we gain the general case that there exists electromagnetic field, Eqs. (43) and (44), thus, can be represented as

$$\mathbf{j}' = \mathbf{j_0} + \alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}\left\{\left(\frac{1}{\sqrt{1-\frac{(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu})^2}{\mathbf{c}^2}}}-1\right)\frac{\mathbf{j_0}\cdot\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}}{\mathbf{c}^2} - \frac{\rho_\mathbf{g}}{\sqrt{1-\frac{(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu})^2}{\mathbf{c}^2}}}\right\}, \tag{46}$$

$$\rho' = \frac{\rho_g - \frac{\mathbf{j_0}\cdot\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu}}{c^2}}{\sqrt{1-\frac{(\alpha_{\mathbf{A}_\mu}\mathbf{J}_{\mathbf{A}_\mu})^2}{c^2}}}, \tag{47}$$

and the corresponding consistent condition is $\left|\alpha_{\mathbf{A}_\mu}\right| < \frac{c}{J_{A_\mu}}$. When $\alpha_{\mathbf{A}_\mu}$ is generally chosen as along the motion direction $\mathbf{e_x}$, and J_{A_μ} is taken as magnetic field B, we, thus, can have

$$j_x' = \frac{j_{0x} - \rho_g \alpha B}{\sqrt{1-\frac{(\alpha B)^2}{c^2}}}, \tag{48}$$

$$\rho' = \frac{\rho_g - \frac{\alpha B}{c^2} j_{0x}}{\sqrt{1-\frac{(\alpha B)^2}{c^2}}}, \tag{49}$$

where α is a small parameter determined by the nuclear physical experiments under the external magnetic field B.

In order to test the theory, considering the case of $j_{0x} = 0$, in Eq. (49) we have

$$\rho' = \frac{\rho_g}{\sqrt{1 - \frac{(\alpha B)^2}{c^2}}}. \tag{50}$$

Because α is the coupling parameter, Eq. (50) shows the relation of density ρ's coupling effect with external magnetic field, which conforms well to Chakrabarty *et al.* (1997)'s research about dense nuclear matter in a strong magnetic field.

4. DIFFERENT MASS SPECTRUM ABOUT DIFFERENT DYNAMICAL BREAKING AND VACUUM BREAKING

Because σ_0 and $\boldsymbol{\pi}_\mathbf{0}$ may be made from the condensations of fermion-antifermion pairs, we can discuss the concrete expressions of different mass spectrum about dynamical breaking and spontaneous vacuum symmetry breaking as follows:

(i) When considering the following dynamical breaking

$$\langle\bar{\psi}(x)\psi(x)\rangle_0^J\ |_{J=0} \neq 0,\ \ \langle\bar{\psi}(x)\gamma_5\boldsymbol{\tau}\psi(x)\rangle_\mathbf{0}^\mathbf{J}\ |_{\mathbf{J}=\mathbf{0}} = \mathbf{0}, \tag{51}$$

we have

$$\boldsymbol{\pi}_\mathbf{0} = \mathbf{0},\ \ \lambda\sigma_\mathbf{0}(\nu^\mathbf{2} - \sigma_\mathbf{0}^\mathbf{2}) = g\langle\bar{\psi}(x)\psi(x)\rangle_\mathbf{0}^\mathbf{J}\ |_{\mathbf{J}=\mathbf{0}} = -gtrS_F(\mathbf{0}), \tag{52}$$

the corresponding spontaneous vacuum symmetry breaking is

$$\sigma(x) \longrightarrow \sigma(x) + \sigma_0, \tag{53}$$

the Lagrangian density, thus, is

$$\begin{aligned}\mathfrak{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \bar{\psi}(x)[\gamma^\mu(\partial_\mu - ieA_\mu) + m_f]\psi(x) - g\bar{\psi}(x)[\sigma(x) \\ &+ i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(x)\gamma_\mathbf{5}]\psi(x)] - \frac{1}{2}(\partial_\mu\sigma(x))^2 - \frac{1}{2}m_\sigma^2\sigma^2(x) \\ &- \frac{1}{2}(\partial_\mu + ieA_\mu)\boldsymbol{\pi}^+(x)\cdot(\partial_\mu - ieA_\mu)\boldsymbol{\pi}(x) - \frac{\mathbf{1}}{\mathbf{2}}m_\pi^\mathbf{2}\boldsymbol{\pi}^\mathbf{2}(x) \\ &- \frac{\lambda}{4}(\sigma^2(x) + \boldsymbol{\pi}(x))^\mathbf{2} - \lambda\sigma_\mathbf{0}\sigma(x)(\sigma^\mathbf{2}(x) \\ &+ \boldsymbol{\pi}^\mathbf{2}(x)) - gtrS_F(\mathbf{0})\sigma(x).\end{aligned} \tag{54}$$

One obtains that the fermion doublet masses are

$$m_f = g\sigma_0. \tag{55}$$

Masses of $\sigma(x)$ and $\boldsymbol{\pi}(x)$, respectively, are

$$m_\sigma^2 = \lambda(3\sigma_0^2 - \nu^2) = 2\lambda\sigma_0^2 + gS_F(0)/\sigma_0, \tag{56}$$

$$m_\pi^2 = \lambda(\sigma_0^2 - \nu^2) = gtrS_F(0)/\sigma_0. \tag{57}$$

Thus, when there is no dynamical breaking, we obtain $\sigma_0^2 = \nu^2$, which just shows ν^2's physical meaning, i.e., σ_0 is, in this case, just the spontaneous vacuum breaking parameter, and $m_\pi^2 = 0$. Even so, σ particle and fermions acquire masses, namely $m_\sigma^2 = 2\nu^2$, $m_f = g|\nu|$. Therefore, the masses of σ particle and fermion doublet naturally come from only the vacuum breaking. In general case, when there exist both dynamical breaking and spontaneous vacuum breaking, not only $\boldsymbol{\pi}$ meson and fermions gain masses, but also σ and $\boldsymbol{\pi}$ masses are not equal. More generally, we may take $\sigma_0' = \langle\bar{\psi}(x)\psi(x)\rangle_0^J$ in which σ_0' is the running spontaneous vacuum breaking value. This result means that σ_0' is the excited state, which make fermion doublet, σ particle and $\boldsymbol{\pi}$ gain effective masses relative to different external sources.

(ii) when $\sigma_0 = 0$, $\pi_0 = 0$, analogous to the research about Eqs. (50) and (57), we get $\sigma(x)$ and $\pi(\mathbf{x})$ meson having the same mass (Itzykson and Zuber, 1980)

$$m_\sigma^2 = m_\pi^2 = -\lambda\nu^2. \tag{58}$$

Further using Eq. (55) in the cases of $\sigma_0 = 0$ and $\pi_0 = 0$, we obtain the fermion doublet keeping no mass.

(iii) General dynamical breaking

We now consider a general dynamical breaking. From Eqs. (34) and (35) we see that

$$\sigma_0 = Kg\langle\bar{\psi}(x)\psi(x)\rangle_0^J\ |_{J=0} \neq 0, \boldsymbol{\pi}_{\mathbf{0}} = iKg\langle\bar{\psi}(x)\gamma_{\mathbf{5}}\boldsymbol{\tau}\psi(x)\rangle_{\mathbf{0}}^J\ |_{J=\mathbf{0}} \neq \mathbf{0}. \tag{59}$$

Then the corresponding spontaneous vacuum symmetry breaking is

$$\sigma(x) \longrightarrow \sigma(x) + \sigma_0,\ \boldsymbol{\pi}(x) \longrightarrow \boldsymbol{\pi}(x) + \varepsilon\boldsymbol{\pi}_{\mathbf{0}}, \quad \mathbf{0} \leq \varepsilon \leq \mathbf{1}, \tag{60}$$

where ε is a running breaking coupling parameter determined by different physical experiment data.

Because electromagnetic interaction is very weaker than strong interaction, electromagnetic interaction may be neglected. The corresponding Lagrangian, density is

$$
\begin{aligned}
\mathfrak{L} = & -\bar{\psi}(x)[\gamma^{\mu}\partial_{\mu} + m_f]\psi(x) - g\bar{\psi}(x)[\sigma(x) + i\boldsymbol{\tau}\cdot\boldsymbol{\pi}(\mathbf{x})\gamma_5]\psi(x) \\
& - \frac{1}{2}(\partial_{\mu}\sigma(x))^2 - \frac{m_{\sigma}^2}{2}\sigma^2(x) - \frac{1}{2}(\partial_{\mu}\boldsymbol{\pi}(x))^2 \\
& - \frac{\lambda}{2}[(\sigma_0^2 + \varepsilon^2\boldsymbol{\pi}_0^2 - \nu^2)\boldsymbol{\pi}^2 + \mathbf{2}(\varepsilon\boldsymbol{\pi}_0\cdot\boldsymbol{\pi})^2] - \frac{\lambda}{4}(\sigma^2(x) + \boldsymbol{\pi}^2(x))^2 \\
& - \lambda(\sigma_0\sigma(x) + \varepsilon\boldsymbol{\pi}_0\cdot\boldsymbol{\pi}(x))(\sigma^2(x) + \boldsymbol{\pi}^2(x)) \\
& -2\lambda\sigma_0(\varepsilon\boldsymbol{\pi}_0\cdot\boldsymbol{\pi}(x))\sigma(x) - \frac{\lambda}{2}(\sigma_0^2 + \varepsilon^2\boldsymbol{\pi}_0^2 - \nu^2)(\sigma_0\sigma(x) \\
& +\varepsilon\boldsymbol{\pi}_0\cdot\boldsymbol{\pi}(x)) - \frac{\lambda}{4}(\sigma_0^2 + \varepsilon^2\boldsymbol{\pi}_0^2 - \nu^2)^2,
\end{aligned}
\tag{61}
$$

where masses of the fermions and σ particle , respectively, are

$$m_N = g(\sigma_0 + i\varepsilon\tau\cdot\pi_0\gamma_5), \tag{62}$$

$$m_{\sigma}^2 = \lambda\left(3\sigma_0^2 + \varepsilon^2\pi_0^2 - \nu^2\right). \tag{63}$$

Because of

$$(\pi_0\cdot\pi)^2 = \pi_0^2\pi^2 + \sum_{\substack{i,j=1\\ i\neq j}}^{3}\left(\pi_{i0}\pi_{j0}\pi_i\pi_j - \pi_{i0}^2\pi_j^2\right). \tag{64}$$

Under the condition of

$$\sum_{\substack{i,j=1\\ i\neq j}}^{3}\pi_{i0}\pi_{j0}\pi_i\pi_j = \sum_{\substack{i,j=1\\ i\neq j}}^{3}\pi_{i0}^2\pi_j^2,$$

we obtain $\boldsymbol{\pi}$ meson mass expression

$$m_{\pi}^2 = \lambda\left(\sigma_0^2 + 3\varepsilon^2\boldsymbol{\pi}_0^2 - \nu^2\right). \tag{65}$$

When $\boldsymbol{\pi}_0 = \mathbf{0}$ or $\varepsilon = 0$, the results (iii) are simplified into the results (i) above.

When there is pseudoscalar condensation $\langle\bar{\psi}(x)\tau\gamma_5\psi(x)\rangle_0^{\mathbf{J}}\,|_{\mathbf{J}=\mathbf{0}}$, because the scalar condensation is stronger than the pseudoscalar condensation, the σ_0 is not equal to zero under existing pseudoscalar condensation.

From the above discussion, we may see what no needing Higgs particle, we naturally gain both fermion's masses and boson's (σ and $\boldsymbol{\pi}$) masses. The mechanisms of gaining masses are direct and useful for constructing the weak-electric standard model without Higgs fields. For making fermions and bosons in the other models acquire masses, people usually use the too many adjusting parameters to fit with the physical experiments in the usual unified models, which makes the prediction of the models decrease. We generally deduce that the masses of nucleons, σ and $\boldsymbol{\pi}$ have the effects coming from interactions with external source. It can be seen that σ and π^0 may be made from the different condensations of fermions and antifermions. This leads to that σ and π^0 without electric charge have electromagnetic interaction effect coming from their internal structure. Using the general research of this paper, we can very more study the interactions between nuclei in general situation, these will be written in the other papers.

5. SUMMARY AND CONCLUSION

We consider a general $SU(2)_L \times SU(2)_R \times U(1)_{EM}$ sigma model with external sources, dynamical breaking and spontaneous vacuum symmetry breaking. We present the general basic formulation of the model. This paper founds the different condensed effects about fermions and antifermions, in which the concrete scalar and pseudoscalar condensed expressions of σ_0 and $\boldsymbol{\pi_0}$ bosons are shown up. We have shown that σ and π^0 may be made from the different condensations of fermions and antifermions. We have discovered that σ and π^0 without electric charge have electromagnetic interaction effect coming from their internal structures, which is similar to neutron. A general Lorentz transformation relative to external sources $J_{\text{gauge}} = (J_{A_\mu}, J_{A_\mu^\kappa})$ is derived, using the general Lorentz transformation and four-dimensional currents of nuclear matter (of ground state) with $J_{\text{gauge}} = 0$ we deduced the four-dimensional general relations of different currents of nuclear matter with $J_{\text{gauge}} \neq 0$ relative to the nuclear matter ground state with $J_{\text{gauge}} = 0$, and give the relation of the scalar density's coupling effect with external magnetic field. This result conforms well to the result of Chakrabarty *et al.* (1997) about dense nuclear matter in a strong magnetic field. We also get the useful expressions of different mass spectra about dynamical breaking and spontaneous vacuum breaking. This paper gives running spontaneous vacuum breaking value σ_0' in terms of the external source technique in quantum field theory, and obtains spontaneous vacuum symmetry breaking based on the σ_0', which make nucleon fermion doublet, σ and $\boldsymbol{\pi}$ particles gain effective masses relative to external sources. We have found out the mechanisms of mass production of nucleon fermion doublet and bosons (σ and $\boldsymbol{\pi}$). The mechanism is useful for constructing the unified weak-electric model without fundamental scalar fields. The effect of external sources and nonvanishing values of the scalar and pseudoscalar condensations are given in the present theory, we generally deduce that the masses of nucleons, σ and π partly come from the interactions with different external sources.

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